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Self-Organized Superfluid States in Gravity and Heat Flow

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When ^{4}He is heated from above near the superfluid transition, there can appear a self-organized region, either in normal fluid or superfluid, with the temperature gradient equal to the transition temperature gradient. When it is in a superfluid state, there can be two regimes. Regime M is realized relatively far from the superfluid transition, where thermal resistance due to vortices is described in terms of the conventional Gorter-Mellink mutual friction. In regime G vortices are densely generated, where the line density in units of ξ^{-2} is much larger than in any other previous experiments. Such a self-organized superfluid can coexist with a normal fluid or a superfluid containing only a small number of vortices in a dynamical steady state.

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1. INTRODUCTION

Recently much attention has been paid to the superfluid transition under gravity and heat flow[1−10]. In particular, if heat flow is applied in the direction of gravity, intriguing steady states are realized, in which the temperature gradient ∇T and the pressure-induced gradient $\nabla T_\lambda(p)$ of the transition temperature spontaneously become equal, resulting in a constant reduced temperature $T/T_\lambda(p) - 1 = \varepsilon_c$. Such a state has been called a *self-organized critical state*[11,12]. However, criticality is not reached in ^{4}He in this geometry, to be precise. Therefore, it will be simply called a *self-organized state* in this paper. Here ε_c can be either positive for self-organization in a normal fluid or negative for that in a superfluid. (i) In a self-organized normal fluid state, ε_c cannot be smaller than a lower bound $\varepsilon_T \sim 10^{-8}(g/g_{earth})^{1/(2\nu+x_\lambda)}$ determined by gravity acceleration g relative to that g_{earth} on earth[6]. Here $\nu \cong 2/3$ is the critical exponent of the correlation length ξ and $x_\lambda \cong 0.45$ is the critical exponent of the thermal conductivity λ. This is because the

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critical fluctuations are nonlinearly suppressed by the temperature gradient and λ levels off as $T/T_\lambda - 1$ approaches ε_T. (ii) In a self-organized superfluid, vortices spontaneously appear to achieve the temperature balance. The line density of vortices must then be very high, if space is measured in units of ξ, close to the λ point. Such defects have been observed in 1D simulations of model F as phase slip centers[6,10]. In their experiment, Moeur *et al.*[8] realized a self-organized region that spanned both the normal fluid and superfluid phases.

In this paper we will clarify the scenario of self-organization in superfluids. We will present some simulation results, though still in 1D, to gain insights into this highly nonlinear problem.

2. Superfluid states near the λ point under heat flow

First we consider ^{4}He below the superfluid transition with thermal counterflow, while neglecting gravity. It is well-known that small thermal resistance arises from vortices in the presence of heat flux Q. In a one-dimensional geometry the resultant temperature gradient has been expressed in the Gorter and Mellink form[13],

$$s\frac{d}{dx}T = -A(T)\rho_n w^3 = -A(T)\rho_n (Q/Ts\rho_s)^3, \tag{1}$$

where w is the relative velocity between the superfluid and normal fluid components, s is the entropy per unit mass, and $\rho_s = \rho - \rho_n$ is the superfluid density. The coefficient $A(T)$ is of order $(m_4/\hbar\rho E_0^2)B$, where m_4 is the ^{4}He mass, $E_0 = \ln(R/\xi)$ is a logarithmic factor expected to be considerably larger than 1 with R being the inter-vortex distance, and B is the Hall-Vinen mutual friction coefficient[14]. When the reduced temperature $\tau = T/T_\lambda - 1$ is very small, we have $B \sim (m_4/\hbar)L$, where L is the kinetic coefficient for the complex order parameter[15]. Above T_λ the critical behavior of L is given by ξ/λ, where $\xi = \xi_0\tau^{-\nu}$ is the correlation length and $\lambda = \lambda^*\tau^{-x_\lambda}$. We assume the same critical exponent for L below T_λ to obtain

$$A(T) \propto |\tau|^{x_\lambda - \nu}, \tag{2}$$

Then (1) is written as

$$\frac{1}{T_\lambda}\frac{d}{dx}T = -B_v|\tau|^{-m_v}Q^3, \tag{3}$$

where $m_v = 4\nu - x_\lambda$. Previous experiments were fairly consistent with the above form[16,17]. For example, Ahlers' result for $T_\lambda - T \gtrsim 10^{-4}$ K was fitted to (3) with $m_v = 2.23$ and $B_v = 5 \times 10^{-29}$ in cgs units[16].

Next we consider a characteristic reduced temperature τ_Q in heat flow. In thermal counterflow we have $\psi \propto \exp(-ikx)$, where the wave number k is related to the superfluid velocity as $v_s = \hbar k/m_4$. The heat flux Q is then expressed as

$$Q = \rho s T_\lambda |v_n| \cong s T_\lambda \rho_s |v_s| = (\hbar s T_\lambda/m_4)\rho_s k, \tag{4}$$

where $\rho_s = \rho_s^* |\tau|^\nu$ is the superfluid density. Thus $k \propto Q|\tau|^{-\nu}$. It is known that nonlinear heat flow effects are significant when k becomes of order ξ^{-1}. We thus introduce a crossover correlation length ξ_Q and reduced temperature τ_Q by setting $\xi_Q^{-1} = \xi_{+0}^{-1}\tau_Q^\nu = k \propto Q\tau_Q^{-\nu}$ with $\xi_{+0} = 1.4\text{Å}$[1,3]. At SVP we have

$$\tau_Q = (m_4\xi_{0+}/\hbar s T_\lambda \rho_s^*)^{1/2\nu} Q^{1/2\nu} \cong 0.45 \times 10^{-8} Q^{0.75}, \tag{5}$$

$$\xi_Q = (\hbar s T_\lambda \rho_s^* \xi_{0+}/m)^{1/2} Q^{-1/2} \cong 5.1 \times 10^{-3} Q^{-0.5}\text{cm}. \tag{6}$$

In superfluids the physical quantities are little affected by heat flow for $|\tau| \gg \tau_Q$, while superfluidity itself is broken for $|\tau| \lesssim \tau_Q$.

It is then instructive to rewrite (3) in terms of τ_Q in the following scaling form,

$$\frac{d}{dx}|\tau| = A_v \frac{|\tau|}{\xi}(\tau_Q/|\tau|)^{6\nu}, \tag{7}$$

where

$$A_v = B_v \xi_{+0} A_Q^{-6\nu} |\tau|^{5\nu-1-m_v} = (\rho/sT)(\hbar/m_4)^3 \xi_{+0}^{-2} A(T)|\tau|^{2\nu-1} \tag{8}$$

with $A_Q = (m_4\xi_{0+}/\hbar s T_\lambda \rho_s^*)^{1/2\nu}$ and $\xi \equiv \xi_{+0}|\tau|^{-\nu}$. The dimensionless number A_v is of order $B|\tau|^{1/3}/E_0^2$ and is much smaller than 1. In fact Ahlers' result[16] gives $A_v \sim 1.1 \times 10^{-3}$. Hereafter we neglect the weak temperature dependence of A_v and treat it as a small number of order 10^{-3} for the mathematical simplicity. Taking the origin of the x axis appropriately inside the cell, we may integrate (7) in the form,

$$|\tau(x)|^{5\nu} = |\tau(0)|^{5\nu} + 5\nu A_v(\tau_Q^{6\nu}/\xi_{0+})x. \tag{9}$$

The characteristic length over which the reduced temperature τ changes significantly due to the mutual friction is given by $\ell_M \sim 10^3(|\tau|/\tau_Q)^{6\nu}\xi$. If there is a HeI-HeII interface at $x = 0$, τ on the superfluid side is a negative constant of order $-\tau_Q$ in the region $0 < x < \ell_M \sim 10^3\xi_Q$ within which vortices may be neglected[2]. In addition let us consider the vortex line density n_v in superfluid states. In the original theory[14] the right hand side of (1) is of order $(\hbar/m_4)Bn_v w$, so it is estimated as

$$n_v\xi^2 \sim E_0^{-2}(\tau_Q/|\tau|)^{2\nu}, \tag{10}$$

where the coefficient E_0^{-2} is expected to be small ($\sim 10^{-2}$). Thus $n_v\xi^2$ is very small for $|\tau| \gg \tau_Q$.

3. Regime M in ^{4}He heated from above

Next we include gravity, which can drastically modify the above results when heat flow is applied from above very close to the λ point. We introduce the local reduced temperature ε as

$$\varepsilon = \left[T - T_\lambda(p)\right] \Big/ T_\lambda(p) = (T/T_{\lambda bot} - 1) + G(x - h). \tag{11}$$

The x axis is taken in the downward direction, $T_{\lambda bot}$ is the transition temperature at the bottom ($x = h$), and $G \cong 0.6 \times 10^{-6}\mathrm{cm}^{-1}$ on earth. In a superfluid region with $\varepsilon < 0$, (3) is changed as

$$\frac{d}{dx}|\varepsilon| = -G + B_v|\varepsilon|^{-m_v}Q^3. \tag{12}$$

We notice that there are two cases. In *regime M*, where the mutual fiction is dominant, the right hand side of (12) is positive and $|\varepsilon|$ increases in the downward direction. On the other hand, in *regime G* it is negative and the gravity-induced gradient is dominant.

Let us assume that a normal fluid is in a upper region $0 < x < x_{int}$ and a superfluid is in the lower region $x_{int} < x < h$. If τ_Q is much larger than the characteristic reduced temperature $\tau_g = (\xi_{+0}G)^{1/(1+\nu)}(\cong 10^{-9}$ on earth), the interface structure is determined by heat flow and the reduced temperature on the superfluid side is determined by Q as[1,2,4]

$$\varepsilon = -\tau_\infty = -R_\infty \tau_Q, \tag{13}$$

where R_∞ is a universal number and is expected to larger than 2. The superfluid region is totally in regime M if the right hand side of (12) is positive as $x \to +x_{int}$ or

$$\xi_{+0}G < A_v R_\infty^{1-5\nu} \tau_Q^{1+\nu}. \tag{14}$$

Here we set $R_\infty \sim 2$; then, the above relation becomes

$$\tau_Q \gtrsim 10^3 \tau_g \quad \text{or} \quad Q \gtrsim 10^3 (g/g_{earth})^{2\nu/(1+\nu)} \quad (\mathrm{erg/cm^2 s}). \tag{15}$$

The temperature profile in regime M is exemplified by the curves 1 and 2 in Fig.1. If the superfluid region is sufficiently wide, we notice that ε tends to a limiting value given by

$$\varepsilon_c = -(B_v Q^3/G)^{1/m_v} \sim -A_v^{1/m_v}(\tau_Q/\tau_g)^{(1+\nu)/m_v}\tau_Q. \tag{16}$$

The relaxation length is given by $|\varepsilon_c|/G$. In this self-organized state the

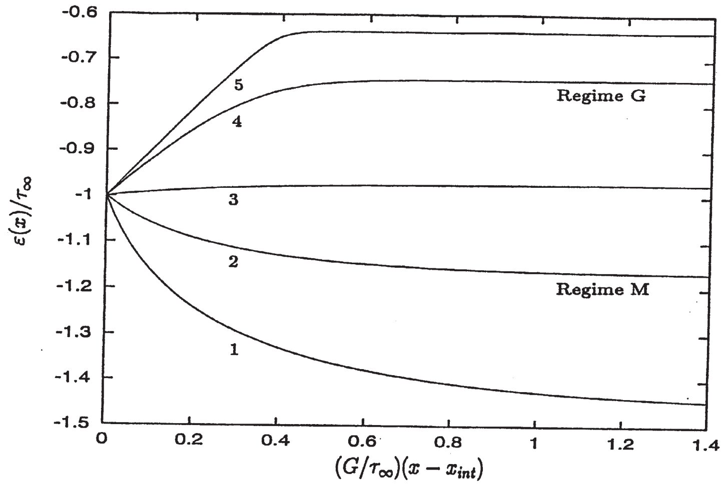

Fig. 1. Schematic profiles of the local reduced temperature ε in a superfluid region on earth with $Q = 2160, 1240, 710, 200$ and 31 erg/sec^{2}s from below. A normal fluid region is in the region $x < x_{int}$ and $\varepsilon = -\tau_\infty$ at $x = 0$. Regime G is realized for $Q \lesssim 10^3$ erg/cm^{2}s on earth in this geometry.

temperature gradient due to defects becomes equal to the transition temperature gradient:

$$\left(\frac{d}{dx}T\right)_{defect} = \frac{d}{dx}T_\lambda. \tag{17}$$

The reduced temperature at the bottom ε_{bot} turns out to be larger than $10^3\tau_g (\sim 10^{-6}$ on earth) in magnitude.

4. Regime G in ^{4}He heated from above

Much more puzzling is *regime G*, where the reverse condition of (14) or (15) holds and $|\varepsilon_{bot}| \lesssim 10^3\tau_g$. In this case the system approaches the λ point with constant Q with increasing the distance from the interface. The dimensionless wave number $K = k\xi$ is related to $|\varepsilon|$ as

$$K(1 - K^2) = (\xi/\xi_Q)^2 = (\tau_Q/|\varepsilon|)^{2\nu}, \tag{18}$$

where $\xi = \xi_{+0}|\varepsilon|^{-\nu}$ is the local correlation length. Thus K increases up to a critical value K_c for $x - x_{int} \gtrsim \ell_{GQ}$, where $\ell_{GQ} = \epsilon_\infty/G \sim 10^{-2}Q^{1/2\nu}(g_{earth}/g)$ cm. Hereafter we set $K_c = 1/\sqrt{3}^{18}$. Here vortices should be densely proliferated to produce enhanced mutual friction. Notice that the Gorter-Mellink mutual friction expression is applicable only under the weak flow condition $K \ll 1$. It is worth noting that Clow and Reppy[19] measured the critical superfluid velocity v_{sc}, above which vortex resistance becomes appreciable, for ^{4}He near the superfluid transition in porous materials. Their data yield $K_c = m_4 v_{sc}\xi/\hbar = 0.062$, which is one order of magnitude smaller than $1/\sqrt{3}$. Therefore, our situation in which $K \sim 1/\sqrt{3}$ is very unusual and new.

Because the free energy to create a vortex line is decreased with increasing K, the vortex line density n_v will be determined by a generalized Vinen equation[14],

$$\frac{d}{dt}n_v = A_1 w n_v^{3/2} - A_2(1-3K^2)^{\gamma_v} n_v^2, \tag{19}$$

where A_1 and A_2 are appropriate constants and the factor $(1-3K^2)^{\gamma_v}$ has been newly introduced. Theory for the exponent γ_v has not yet been constructed. In steady states we have $n_v \propto w^2/(1-3K^2)^{2\gamma_v}$ and generalize (12) as

$$\frac{d}{dx}|\varepsilon| = -G + B_v|\varepsilon|^{-m_v}Q^3(1-3K^2)^{-2\gamma_v}, \tag{20}$$

The last factor accounts for the growing mutual friction as $K \to 1/\sqrt{3}$. We solve the above equation at $\gamma_v = 1$ and $R_\infty = 2$ to obtain the temperature curves 3-5 in Fig.1. We recognize that the system again tends to a self-organized superfluid state for $x - x_{int} \gtrsim \ell_{GQ}$, in which the balance (17) is attained and $|\varepsilon|$ approaches a limiting reduced temperature ε_c. If $Q \ll 10^3(g/g_{earth})^{2\nu/(1+\nu)}$, K must be close to $1/\sqrt{3}$ and $\varepsilon \to \varepsilon_c \cong -2\tau_Q$.

5. Numerical analysis in regime G

To perform numerical analysis we propose a local equilibrium renormalized model. We measure the reduced temperature in units of a reference reduced temperature $\tilde{\tau}$. Here we set $\tilde{\tau} = 2.5 \times 10^{-8}$. The space is then measured in units of the corresponding correlation length $\tilde{\xi} = 1.6 \times 10^{-3}$ cm. The dimensionless gravity coefficient $\tilde{G} = G\xi_{+0}/\tilde{\tau}$ becomes 0.04 on earth. We propose the dynamic equations[6],

$$\frac{\partial}{\partial t}\Psi = ia^{-1}A\Psi - L\left[\varepsilon\xi^{-1/2} - \nabla^2 + \xi^{-1}|\Psi|^2\right]\Psi, \tag{21}$$

$$\frac{\partial}{\partial t}M = a\mathrm{Im}(\Psi^*\nabla^2\Psi) + \nabla\cdot\lambda\nabla A, \tag{22}$$

where Ψ is the scaled complex order parameter, $A = (T - T_{\lambda bot})/T_\lambda\tilde{\tau}$, $\varepsilon = A + \tilde{G}(x-h)$ (in the same notation as in (11)), and

$$M = A - \frac{1}{2}a^2\xi^{-1/2}|\Psi|^2 \tag{23}$$

is the scaled entropy deviation. The thermal noise terms are omitted for simplicity. The coefficients in this model are obtained by appropriate scaling of those of model F renormalized at the local correlation length ξ. In gravity ξ should not exceed the characteristic length $\ell_g = \tilde{G}^{-\nu/(1+\nu)} (= 3.62$ on earth in units of $\tilde{\xi})$, so that we define the local correlation length as $\xi = \ell_g \tanh(1/\ell_g|\varepsilon|^{2/3})$. The scaled kinetic coefficients behave as $\lambda = b_\lambda \xi^{0.675}$ and $L = b_\psi \xi^{0.325}$, where b_λ and b_ψ are of order 1. The ratio $w = L/\lambda$ is known to be considerably smaller than 1 in magnitude as $T \to T_\lambda$[20].

Numerical investigation of self-organized superfluid states in regime G was performed in Ref.[6] (but the renormalization factor $\xi^{-1/2}$ in (24) was not present). There, we prepared a normal fluid state for $t < 0$ and lowered the bottom end below the transition at $t = 0$. Then a superfluid region appeared from the bottom and expanded upwards. In such an expanding superfluid in regime G, dense defects (phase slip centers in 1D) were generated such that the balance (17) was attained. In real ^{4}He in this geometry, an expanding superfluid region is in regime G only when the reduced temperature $\varepsilon_b (< 0)$ at the bottom is smaller than $10^3 \tau_g$ in magnitude. For deeper quenching regime M will be realized.

In this paper we prepare an equilibrium superfluid state for $t < 0$ and subtract a constant heat flux Q_{bot} from the bottom for $t > 0$ in the geometry of Ref.[8]. At sufficient development of the new phase from below, we apply a heat flux Q_{top} from the top. If $Q_{top} = Q_{bot} = Q$ we can realize a dynamical steady state. Figs.2-4 display the profiles of $T/T_{\lambda bot} - 1$, ε, and ρ_s obtained in 1D simulation of the above model with $a = b_\lambda = 1$ and $b_\psi = 0.2$. In Fig.2 we can see a self-organized normal fluid state in the lower region. As Q is increased, however, the thermal conductivity λ saturates and the bottom end becomes unstable against formation of a superfluid embryo. As shown in Fig.3, at $Q \sim 1$ erg/cm^{2}s, we can see that superfluid domains appear periodically and travel upwards. They behave as *solitons* with negative excess entropy (like holes in electric current in metals). Once such moving domains appear, the space average of ε in the self-organized region becomes negative. This indicates that the transition between self-organized normal and soliton-carrying states is weakly discontinuous. We cannot decide whether this picture is applicable in 3D or not. In Fig.4 coexistence of a self-organized superfluid region with defects and a superfluid region without defects is shown. In this case there is no sharp boundary between the two phases and the width of the transition region is on the order of the defect spacing. Here continuous generation and annihilation of defects occur in the self-organized region. Second sounds are then emitted into the upper superfluid region, causing large scale temperature perturbations. In Fig.5 we show the average reduced temperature ε_c in the self-organized region vs

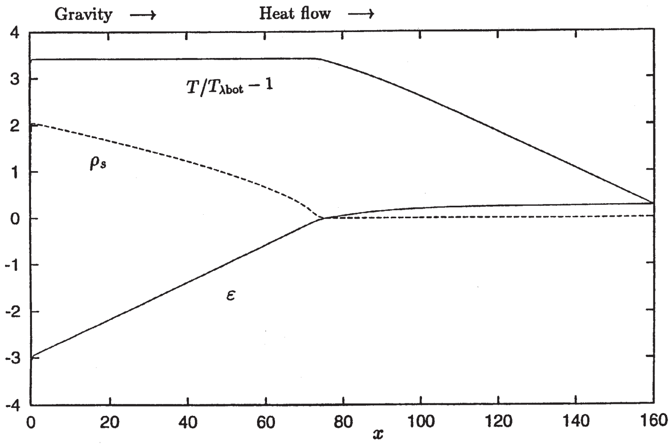

Fig.2. The reduced temperatures $T/T_{\lambda bot} - 1$ and ε (in units of 2.5×10^{-8}) (solid lines) and the superfluid density (broken line) in a steady state at $Q = 0.77$ erg/cm^{2}s, in which the lower part ($x \gtrsim 85$) is a self-organized normal fluid and the upper part is a superfluid. The space is scaled in units of 1.6×10^{-3} cm.

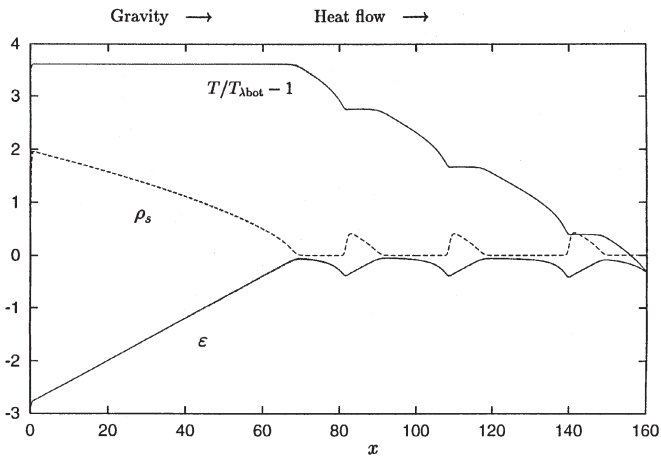

Fig.3. A self-organized state at $Q = 1.6$ erg/cm^{2}s, which is marginal between self-organized normal fluid and superfluid. Here soliton-like superfluid domains are slowly traveling upwards at a constant speed.

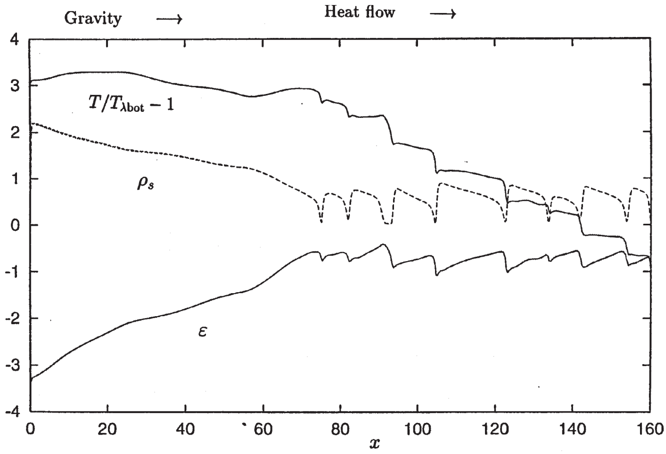

Fig.4. A self-organized superfluid with defects below a superfluid without defects in regime G at $Q = 11$ erg/cm^{2}s applied from above.

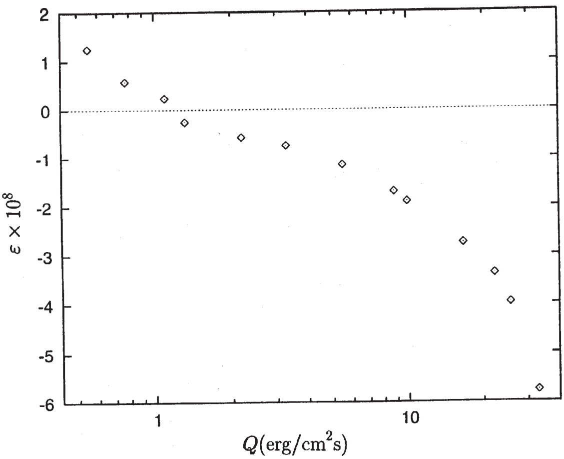

Fig.5. Average reduced temperature (in units of 10^{-8}) vs heat flux Q (erg/cm^{2}s) in dynamical steady states in the geometry of Ref.[8] obtained from 1D simulation of the model (21) and (22).

heat flux Q. Here agreement can be seen with the experiment[8], but it will have only qualitative meaning because our simulation is in 1D. We recognize that defects are crucial in self-organized heat transport in superfluids.

6. Summary

We have shown the existence of conceptually new nonequilibrium states exhibiting a kind of defect turbulence. In fact the scaled vortex line density $n_v\xi^2$ in (10) can be of order E_0^{-2} in regime G, while it is very small in regime M. Understanding of the true physical nature of regime G is still primitive. More experiments and theoretical efforts are needed.

REFERENCES

1. A. Onuki, J. Low Temp. Phys. **50**, 433 (1983); **55**, 309 (1984).
2. A. Onuki, Japanese Journal of Applied Physics **26**, 365 (1987).
3. R.V. Duncan, G. Ahlers and V. Steinberg, Phys. Rev. Lett. **60**, 1522 (1988). 219 (1990).
4. R. Haussmann and V. Dohm Phys. Rev. B **46**, 6361 (1992); Z. Phys. B **87**, 229 (1992).
5. A. Onuki and Y. Yamazaki, J. Low Temp. Phys. **103**, 131 (1996).
6. A. Onuki, J. Low Temp. Phys. **104**, 133 (1996).
7. G. Ahlers and F.-C. Liu, J. Low Temp. Phys. **105**, 255 (1996).
8. W.A. Moeur, P.K. Day, F-C Liu, S.T.P. Boyd, M.J. Adriaans and R.V. Duncan, Phys. Rev. Lett. **78**, 2421 (1997).
9. R. Haussmann, Phys. Rev. B **60**, 12349 (1999).
10. P.B. Weichman and J. Miller, J. Low Temp. Phys. **119**, 155 (2000).
11. P. Bak, C. Tang and K. Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987).
12. J. Machta, D. Candela and R.B. Hallock Phys. Rev. E **47**, 4581 (1993).
13. C.J. Gorter and J.H. Mellink, Physica **15**, 285 (1949).
14. H.E. Hall and W.F. Vinen, Proc.Roy.Soc.London A**238**, 204 (1956); W.F. Vinen, ibid. **242**, 493 (1957).
15. A. Onuki, J. Low Temp. Phys. **51**, 601 (1983); Prog.Theor.Phys. **70**, 57 (1983).
16. G. Ahlers, Phys. Rev. Lett. **22**, 54 (1969).
17. P. Leiderer and F. Pobell, J. Low Temp. Phys. **3**, 577 (1970).
18. Note that $K_c = 1/\sqrt{3}$ in mean field theory[19]. We neglect a correction about 10% from the critical fluctuations[4].
19. J.R. Clow and J.D, Reppy, Phys. Rev. Lett. **19**, 291 (1967); J.S. Langer and J.D. Reppy, in *Progress in Low Temperature Physics* **6**, (C. Gorter, ed.) North-Holland, Amsterdam, p.1 (1970).
20. P.C. Hohenberg, Physica B **109** & **110**, 1436 (1982).